Deterministic Chaos Approach to Multistable Perception

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Abstract

Multistable perception is perception in which two (or more) interpretations of the same ambiguous image alternate while an observer looks at them. Perception undergoes involuntary and random-like change. The question arises whether the apparent randomness of alternation is real (that is, due to a stochastic process) or whether any underlying deterministic structure to it exists. In this paper, we examine the spatially coherent temporal behaviors of multistable perception model based on the chaotic neural network from the viewpoint of bottom-up high dimensional approach, aiming at the functioning of chaos in dynamic perceptual processes.

1 Introduction

Multistable perception is perception in which two (or more) interpretations of the same ambiguous image alternate spontaneously while an observer looks at them. Figure-ground, perspective (depth) and semantic ambiguities are well known (As an overview, for example, see [1] and [2]). Actually, when we view the Necker cube which is a classic example of perspective alternation, a part of the figure is perceived either as front or back of a cube and our perception switches between the two different interpretations as shown in Fig.1. In this circumstance the external stimulus is kept constant, but perception undergoes involuntary and random-like change. The measurements have been quantified in psychophysical experiments and it becomes evident that the frequency of the times between such changes $T(n)$ in Fig.2 is approximately Gamma distributed [3, 4, 5].

Although the physiological and anatomical understandings of the effect are unknown, theoretical model approaches to explaining the facts have been made mainly from three situations based on the synergetics [6, 7, 8], the BSB (brain-state-in-a-box) neural network model [9, 10], and the PDP (parallel distributed processing) schema model [11, 12, 13]. Common to these approaches is that top-down designs are applied so that the model can be manipulable by a few parameters and upon this basis fluctuating sources are brought in. The major interests
seem to be not in the relation between the whole function and its element (neuron), but in the model building at the phenomenological level.

Among the models, synergetic Ditzinger-Haken model [6, 7] is known to be able to reproduce the Gamma distribution of $T$ well by subjecting the time-dependent attention parameters to stochastic forces (white noises). However, the question arises whether the apparent randomness of alternation is real (that is, due to a stochastic process) or whether any underlying deterministic structure to it exists.

Until now diverse types of chaos have been confirmed at several hierarchical levels in the real nervous systems from single cells to cortical networks (e.g. ionic channels, spike trains from cells, EEG) [14]. This suggests that artificial neural networks based on the McCulloch-Pitts neuron model [15] should be re-examined and re-developed. Chaos may play an essential role in the extended frame of temporal behaviors of multistable perception model [17], the dynamic learning and retrieving features of the associative memory have been studied [18, 19]. In this paper, we examine the spatially coherent activities arise on the neural network and cause the transitions between stable states of HNP. This situation corresponds to the dynamic multistable perception.

2 Model and Method

The chaotic neural network (CNN) composed of $N$ chaotic neurons is described as [17, 19]

$$X_i(t+1) = f(\eta_i(t+1) + \zeta_i(t+1))$$

$$\eta_i(t+1) = \sum_{j=1}^{N} w_{ij}(t) X_j(t-d)$$

$$\zeta_i(t+1) = -\alpha \sum_{d=0}^{t} k_d^i X_i(t-d) - \theta_i$$

where $X_i$: output of neuron $i$; $\eta_i$ : threshold of neuron $i$, $k_f(k_r)$ : decay factor for the feedback(refractoriness) ($0 < k_f, k_r < 1$), $\alpha$ : refractory scaling parameter, $f$: output function defined by $f(y) = tanh(y/2\epsilon)$ with the steepness parameter $\epsilon$. Owing to the exponentially decaying form of the past influence, Eqs.(2) and (3) can be reduced to

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{j=1}^{N} w_{ij}(t) X_j(t)$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha X_i(t) + a$$

where $a$ is temporally constant $a \equiv -\theta_i(1-k_r)$.

All neurons are updated in parallel, that is, synchronously. The network corresponds to the conventional discrete-time Hopfield network:

$$X_i(t+1) = f \left( \sum_{j=1}^{N} w_{ij} X_j(t) - \theta_i \right)$$

where $\alpha = k_f = k_r = 0$ (Hopfield network point (HNP)). The asymptotical stability and chaos in discrete-time neural networks are theoretically investigated in Refs. [22, 23].

Under external stimuli, Eq.(1) is influenced as

$$X_i(t+1) = f(\eta_i(t+1) + \zeta_i(t+1) + \sigma_i)$$

where $\sigma_i$ is the effective term by external stimuli. This is a simple and unartificial incorporation of stimuli as the changes of neural active potentials.

The two competitive interpretations are embedded in the network as minima of the energy map:

$$E = -\frac{1}{2} \sum_{ij} w_{ij} X_i X_j$$

at HNP. This is done by using a iterative perception learning rule for $p(<N)$ patterns \{$\xi^\mu_i\}$ ($i=1,\ldots,N$) in the form:

$$w_{ij}^{new} = w_{ij}^{old} + \sum_{\mu} \delta w_{ij}^{\mu}$$

with

$$\delta w_{ij}^{\mu} = \frac{1}{N} \theta(1 - \gamma^\mu_i)^{\xi^\mu_i \xi^\mu_j}$$

where $\gamma_i^\mu \equiv \xi^\mu_i \sum_{j=1}^{N} w_{ij}^{\mu} \xi^\mu_j$ and $\theta(h)$ is the unit step function. The learning mode is separated from the performance mode by Eq.(7).

The conceptual picture of our model is shown in Fig.3. Under the external stimulus $\{\sigma_i\}$, chaotic activities arise on the neural network and cause the transitions between stable states of HNP. This situation corresponds to the dynamic multistable perception.

3 Simulations and Results

To carry out computational experiments, we use the $12 \times 13(N=156)$ non-orthogonal 10 random patterns \{$\xi^\nu_i$\} ($\nu=1,\ldots,10, i=1,\ldots,N$) as a set of ambiguous figure stimuli: $\{\sigma_i\} = s\{\xi^\nu_i\}$. $s$ is the strength factor of stimulation. For each of them, two interpretation patterns \{$\xi^{\nu_1}_i\}$ and \{$\xi^{\nu_2}_i\}$ are prepared, as an example, by changing 15 white ($\xi_i = -1$) pixels to black ($\xi_i = +1$) ones which do not overlap between $\nu_1$ and $\nu_2$ as shown by shaded and dotted in Fig.4, and are memorized following the above learning rule ($p=20$).
Figure 3: Conceptual picture illustrating state transitions induced by chaotic activity

Figure 5 shows a time series evolution of CNN\( (k_f = 0.5, k_r = 0.8, \alpha = 0.34, \alpha = 0, \varepsilon = 0.015) \) under the stimulus \( \{\sigma_i\} = 0.7\{\xi_i^1\} \). Here,

\[
m^{11}(t) = \frac{1}{N} \sum_{i=1}^{N} \xi_{i}^{11} X_i(t) \tag{11}
\]

and is called the overlap of the network state \( \{X_i\} \) and the interpretation pattern \( \{\xi_i^{11}\} \). A switching phenomenon between \( \{\xi_i^{11}\} \) \( (m^{11} = 1.0) \) and \( \{\xi_i^{12}\} \) \( (m^{11} = 0.62) \) can be observed. Bursts of switching are interspersed with prolonged periods during which \( \{X_i\} \) trembles near \( \{\xi_i^{11}\} \) or \( \{\xi_i^{12}\} \). Evaluating the maximum Lyapunov exponent [24] to be positive \( (\lambda_1 = 0.26) \), we find that the network is dynamically in chaos. Figure 6 shows the return map of the active potential \( h_i(t) = \eta_i(t) + \zeta_i(t) + \sigma_i \) of a neuron(#12). Its chaotic transient corresponding to the network switching behavior is demonstrated in Fig.7. In the cases \( \lambda_1 < 0 \), such switching phenomena do not arise.

From the \( 2 \times 10^5 \) iteration data (until \( t = 2 \times 10^5 \)) of Fig.5, we get 1257 events staying near one of the two interpretations, \( \{\xi_i^{12}\} \). As can be seen from Fig.8 magnified for t-axis, they have various persistent durations \( T(1) \sim T(1257) \) which seem to have a random time course by the return map in \( (T(n), T(n+1)) \) shown in Fig.9. From the evaluation of the autocorrelation function for \( T(n) \), \( C(k) = < T(n+k)T(n) > - < T(n) > < T(n+k) > \) (here, \( <> \) means an average over time), we find the lack of even short term correlations as shown in Fig.10. This indicates that successive durations \( T(n) \) are independent. The frequency of occurrence
Figure 6: Return map of the active potential $h_i$ of #12 neuron for the data up to $t=5000$. Solid line traces its behavior for a typical $T$ term $t=3139$ to 3244.

of $T$ is plotted for 1257 events in Fig.11. The distribution is well fitted by Gamma distribution

$$G(\tilde{T}) = \frac{b^n\tilde{T}^{n-1}e^{-b\tilde{T}}}{\Gamma(n)}$$  \hspace{1cm} (12)

with $b = 0.918, n = 4.68(\chi^2 = 0.0033, r = 0.98)$, where $\Gamma(n)$ is the Euler-Gamma function. $\tilde{T}$ is the normalized duration $T/15$ and here 15 step interval is applied to determine the relative frequencies.

The results are in good agreement with the characteristics of psychophysical experiments [4, 5, 3]. Similar results to the above example are obtained in the appropriate parameter zone such as in Fig.12 within regions where the network may induce chaotic activities under external stimuli. It is found that aperiodic spontaneous switching does not necessitate some stochastic description as in the synergetic D-H model [6, 7]. In our model, the neural fatigue effect proposed by Köhler[25] is considered to be supported by the neuronal refractiness $\alpha$, and its fluctuation originates in the intrinsic chaotic dynamics of the (high dimensional) network.

4 Conclusion

We have shown that the deterministic neural chaos leads to perceptual alternations as responses to ambiguous stimuli in the chaotic neural network. Its emergence is based on the simple process in a realistic bottom-up high dimensional framework. Our demonstration suggests functional usefulness of the chaotic activity in perceptual systems even at higher cognitive levels. The perceptual alterna-

Figure 7: Chaotic transient on the $h_i$ return map of #12 neuron for several switching $T$ terms.

Figure 8: Time series of the overlap with $\{\xi_i^{11}\}$ under the stimulus $\{\xi_i^1\}$, magnified for t-axis.

Figure 9: Return map of the persistent durations staying one of the two interpretations for the data ($T(1) \sim T(1257)$) in Fig.5 (up to $t = 2 \times 10^5$).
tion appears to be an inherent feature built in the chaotic neuron assembly.

The perceptual switching in binocular rivalry[26, 27, 28] shares many features with multistable perception, and might be the outcome of a general (common) neural mechanism. The matching process between input pattern and stored templates through the cortico-cortical interaction of the lower and higher cortical areas[29] may serve as a candidate of this mechanism. It will be interesting to study the brain with the experimental technique (e.g., fMRI) under the circumstance where the perceptual alternation is running.

References


